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Oscillatory Network Model which Learns a Rhythmic Pattern of an External Signal

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1 Introduction

Human being can feel the rhythm in signals. For example, when he listens to a music, he can ride on the rhythm. When he practices playing a rhythmic pattern, he tries to follow the instructed rhythm. During this practice, he can get a skill to play the rhythm unconsciously. These facts can be explained by assuming the existence of an oscillatory network which receives rhythmic inputs and changes the structure of the network so as to generate the rhythmic pattern. Furthermore, oscillatory components have been reported in many systems in living bodies[1, 2]. Taking these facts into consideration, we discuss learning methods to learn an externally applied rhythmic signal by oscillatory networks. In section 2, a learning rule is proposed when each component oscillator receives a teacher signal with a desired frequency and phase. In section 3, an oscillatory network model which learns an arbitrary periodic signal is proposed.

2 Forcing Oscillation Method

In this section, we propose a learning rule for an oscillatory network when a teacher signal is given to each oscillator.

Consider collective oscillators with unidirectional multiple coupling (Fig.1), the dynamics is assumed to be

$$\dot{\theta}_i = \omega_i + \epsilon \sum_{k \in G_i} \sum_{l} w^l_{ki} R(\theta_i - \psi^l_{ki}, \theta_k) + \epsilon_f F(\theta_i, \tilde{\theta}_i), \quad (1)$$

where θ_i and ω_i are the phase and the intrinsic frequency of ith oscillator, respectively, $\tilde{\theta}_i(t) = \Omega_i t + \tilde{\theta}_i(0)$ and Ω_i are the phase and the frequency of a teacher signal to ith oscillator, respectively, ψ^l_{ij} and w^l_{ij} are the constant phase delay and the strength of each coupling, respectively, G_i is the ensemble of the index of coupled oscillators, $R(\theta_i, \theta_j) \equiv P(\theta_i)Q(\theta_j)$ and $F(\theta_i, \tilde{\theta}_i) \equiv P(\theta_i)Q_f(\tilde{\theta}_i)$ show the effect of a coupled oscillator and of a teacher signal, respectively, $P(\theta_i)$ is the phase shift caused by an external signal, $Q(\theta_i)$ and $Q_f(\tilde{\theta}_i)$ represent an output signal from a component oscillator and a teacher signal, respectively, and ϵ and ϵ_f ($\epsilon \sum_{k,l} w^l_{ki} \ll 1$, $\epsilon_f \ll 1$) are positive constant values indicating the strength of the effect from coupled oscillators and the teacher signal, respectively. If the dynamics of an oscillator is expressed by Hopf normal form and the attraction to the limit cycle is strong, $P(\theta) = \sin(\theta - \psi)$ (ψ: constant) holds. If $\Omega_i = n_i \Omega$ (n_i : integer) and $|\omega_i - \Omega_i| = \mathcal{O}(\epsilon)$, eq.(1) is transformed to the following form by redefining $(\omega_i, \theta_i, \tilde{\theta}_i) \to (n_i\omega_i, n_i\theta_i, n_i\tilde{\theta}_i)$ and applying average theorem,

$$\dot{\theta}_i = \omega_i + \epsilon \sum_{k,l} w^l_{ki} \bar{R}^l_{ki} + \epsilon_f \bar{F}_i, \quad (2)$$

where $\bar{R}^l_{ki} \equiv \bar{R}(\phi_{ki} - \psi^l_{ki})$, $\bar{F}_i \equiv \bar{F}(\tilde{\phi}_i)$, $\bar{R}$ (or $\bar{F}$) is the correlation function between $P(\theta)$ and $Q(\theta)$ (or $Q_f(\tilde{\theta})$), $\phi_{ki} \equiv \theta_k - \theta_i$, $\tilde{\theta}_i(t) = \Omega t + \tilde{\theta}_i(0)$, and $\tilde{\phi}_i \equiv \tilde{\theta}_i - \theta_i$. By changing the parameters, ω_i and w^l_{ki}, we want to obtain $\tilde{\phi}_i = 0$ in the learning phase and to recall the phase relation $\phi_i \equiv \theta_{i+1} - \theta_i = \tilde{\theta}_{i+1} - \tilde{\theta}_i$ with the frequency Ω after learning. For this purpose, we propose a dynamics of parameters as the form,

$$\dot{\omega}_i = \varepsilon(\epsilon_f \bar{F}_i + \epsilon \sum_{k,l} w^l_{ki} \bar{R}^l_{ki})$$

$$\dot{w}^n_{ji} = \varepsilon(\epsilon_f \bar{F}_i - \gamma\epsilon \sum_{k,l} w^l_{ki} \bar{R}^l_{ki}) \bar{R}^n_{ji},$$

where $\varepsilon > 0$ and $\gamma > -1$ are constant values determining the learning rate. The change of the intrinsic frequency is caused by the total effect of input signals and the change of the coupling strength is given by the correlation between the effects of the teacher signal and of the coupled oscillator when $\gamma = 0$. The convergence to the desired phase relation $\tilde{\phi}_i = 0$ in learning phase and the recall of the teacher signal after learning are guaranteed except the possibility to stay at unstable points [3], if (1) $\dot{\tilde{\phi}}_i \simeq 0$ is satisfied, i.e., the difference between ω_i and Ω is not so large, (2) $\forall i = 2, ..., N, \sum_{k,l} \left(R(\phi_{ki} - \psi^l_{ki})\right)^2 \neq 0$, (3) $\dot{\phi}_i = \epsilon \sum_{k,l} w^l_{ki} R(\phi_{ki} - \psi^l_{ki})$ $(i = 1, ..., N)$ has only one stable solution, (4) $F(\phi)$ and $\sin\phi$ have the same sign, and (5) $\forall i = 2, ..., N, \forall \phi^0_{ki} \in \{\phi_{ki} | \sum_{k,l} w^l_{ki} R(\phi_{ki} - \psi^l_{ki}) = 0\}$, $\left|\sum_{k,l} w^l_{ki} R'(\phi^0_{ki} - \psi^l_{ki})\right| > |F'(0)|$.

Figure 2 shows the result of a simulation for two oscillators. The appropriate parameters are obtained in

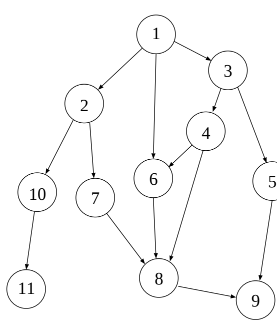

Fig.1. The unidirectional coupling between oscillators. An arrow is composed of several phase delayed connections.

learning phase and fast recovery to the learned phase relation is observed in recalling phase.

3 Oscillatory Network

We propose an oscillatory network which learns a periodic signal. The network model is composed of a layer of mutually coupled oscillators and an output cell (Fig.3). The output cell receives signals $Q_o(\theta)$ from oscillators and outputs the sum of a linear combination of the signals, i.e., the output y is given by $y = \sum_{i,l} g_i^l Q_o(\theta_i - \psi_i^l)$, where ψ_i^l and g_i^l are the phase delay and the weight of each connection, respectively. In learning phase, all oscillators receive a desired rhythmic pattern $Q_f(t)$, i.e., $F_i = P(\theta_i)Q_f(t)$ instead of $F(\theta_i, \tilde{\theta}_i)$ in eq.(1), and the intrinsic frequencies of component oscillators and connections between them are changed according to the proposed learning rule in section 2. Furthermore, connections from each oscillator to the output cell are modulated so as to minimize the square error between the output of the network and the teacher signal, i.e.,

$$\dot{g}_i^k = -\epsilon \frac{\partial E}{\partial g_i^k} = -\epsilon(y - Q_f)Q_o(\theta_i - \psi_i^k). \qquad (3)$$

The simulation result is shown in Fig.4. The output of the network converges to the desired signal in learning phase and the desired signal is recalled after learning from random initial phases of oscillators.

4 Discussion

We proposed a learning method for an oscillatory network. The proposed learning rule for oscillators takes a simple form like Hebbian rule and does not need the back propagation of the error and the derivative of a function but only requires local information.

References

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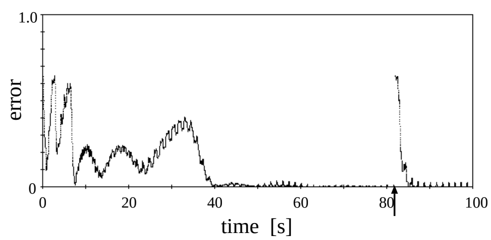

Fig.2. The error during and after learning. The error is defined by the form $E = \sum_{i=1}^{2} \sin^2 \pi(\tilde{\theta}_i - \theta_i)/2$ during learning and $E = \sin^2 \pi(\phi - \tilde{\phi})$ after learning, where $\phi = \theta_2 - 2\theta_1$ and $\tilde{\phi} = \tilde{\theta}_2 - 2\tilde{\theta}_1$. After learning, the simulation is started again from random phases (indicated by an arrow). Functions: $P(\theta) = \sin 2\pi\theta$, $Q(\theta) = (\cos 2\pi\theta + \cos 4\pi\theta)/2$, $Q_f(\tilde{\theta}) = -\cos 2\pi\tilde{\theta}$, $\tau_2 \dot{\bar{R}}^l = -\bar{R}^l + P(\theta_2 - \psi^l)Q(\theta_1)$, $\tau_i \dot{\bar{F}}_i = -\bar{F}_i + P(\theta_i)Q_f(\tilde{\theta}_i)$, $\tau_i = \tau/\omega_i(t)$, $\bar{R}^l(0) = 0$, $\bar{F}_i(0) = 0$, $(i = 1, 2)$. Parameters: $(\Omega_1, \Omega_2) = (0.7, 1.4)$ [Hz], $(\psi_1, \psi_2) = (0, 0.2)$, $\varepsilon = 0.3$, $\tau = 3.0$, $\gamma = 1.0$. Initial condition : $(\theta_1, \theta_2) = (0.3, 0.0)$, $(\tilde{\theta}_1, \tilde{\theta}_2) = (0.8, 0.7)$, $(w^1, w^2) = (0.3, 0.3)^t$, $\omega_i = 1.0$ [Hz]. If $t \le t_0$, $(\epsilon, \epsilon_f) = (0.05, 0.5)$ else $(1.0, 0.2)$, where $t_0 \equiv \min_t\{L(t) \le 0.2\}$, $\tau \dot{L}(t) = -L(t) + \sum_{i=1}^{2} \sin^2 \pi(\tilde{\theta}_i - \theta_i)/2$, $L(0) = 1.0$.

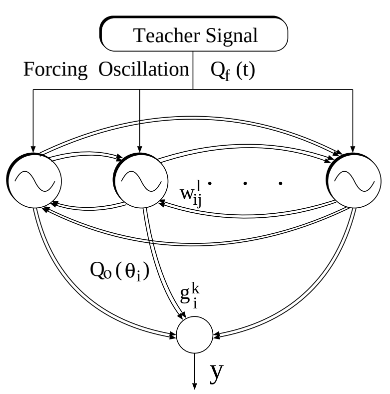

Fig.3. The structure of the proposed oscillator network.

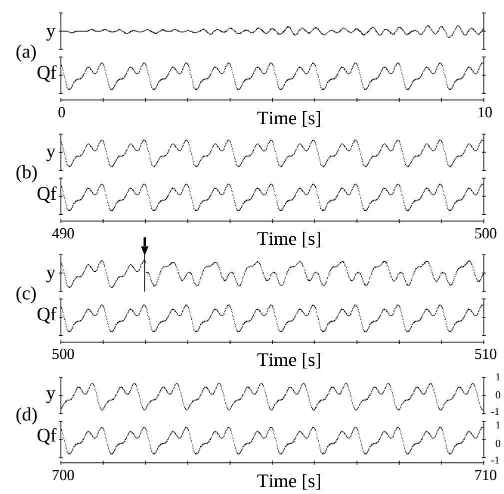

Fig.4. The output of the oscillator-network during and after learning. y is the output of the network and Q_f is the teacher signal. (a)(b) In learning phase. (c)(d) In recalling phase. The arrow indicates the time when the learning is stopped and the network begin to recall the desired signal from random initial phases. During learning, weight values are normalized as $\sum_{jl} |w_{ji}^l| = constant$. Functions: $Q_f(t) = (\cos 2\pi(t + 0.2) + (\cos 4\pi t + \cos 6\pi t)/2)/2$, $Q(\theta) = (\cos 2\pi\theta + \cos 4\pi\theta + \cos 6\pi\theta)/3$, $Q_o(\theta) = \cos 2\pi\theta$, $P(\theta) = \sin 2\pi\theta$. $\bar{F}_i$ and $\bar{R}_{ki}^l$ are calculated by the same manner in Fig.2. Parameters: ϵ, τ, γ, ψ_i are the same as those in Fig.2. If $t \le t_0$, $(\epsilon, \epsilon_f) = (0.2, 0.5)\omega_i$ else $(1.0, 0.1)\omega_i$, where $t_0 \equiv \min_t\{L(t) \le 0.001\}$, $\tau \dot{L}(t) = -L(t) + (Q_f(t) - y(t))^2$, $L(0) = 1.0$. Initial condition: $w_{ji}^k = 0.5/2(N-1)$, $g_i^k = 0$.